07 Game Physics

Tvorba a dizajn počítačových hier (FMFI)
Návrh a vývoj počítačových hier (FIIT)

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3. 11. 2022

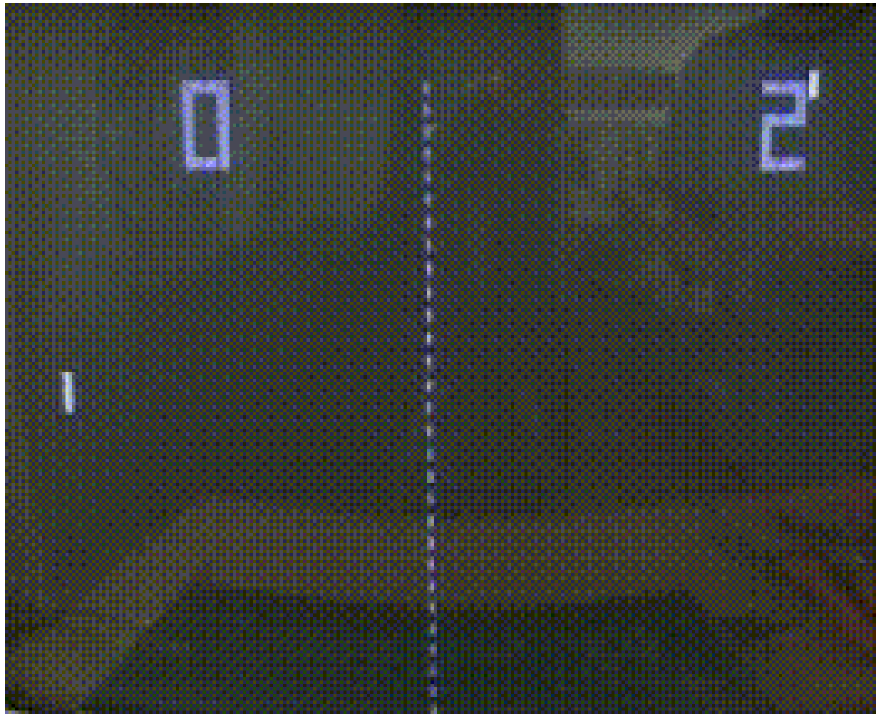
Realistic modeling of the world

- We are mimicking the real world with realistic rendering, environments and animations
- Pre-defined physics animations (exported from a 3D modeling software such as 3ds max) can add physics into our world
- Usually we need to **dynamically** create these animations
- We need to react to user input
- The simplest game physics model: Pong

Image from https://en.wikipedia.org/wiki/Pong

Pong physics model

- We have a ball moving at constant speed
- We render one frame every 20ms -> 50 FPS
- Whenever the ball hits a paddle, we compute new **velocity**
  - That is, direction and speed
- Paddles are moved only by user input, not affected by the ball
- 3D points: $P, Q, R \ldots \in \mathbb{E}^3$
- 3D vectors: $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \ldots \in \mathbb{R}^3$



Pong physics model (2)

- n = current frame, $n+1$ = next frame
- Update the position of the ball every frame
  - $P_{n+1} = P_n + (\boldsymbol{v} \ * \Delta t)$
- Bounce off the top and bottom of the screen
- Detect if a collision occurred between frame n and frame $n+1$
  - Is the line $\overline{P_n P_{n+1}}$ intersecting the paddle line?
- On collision, change the ball's velocity
  - Compute the exact point at which the ball will bounce off
  - Change velocity by reflecting the velocity vector
- End game if the ball intersects the left or right of the screen

Complex physics models

- To perfectly simulate the real world, we would need lots of different physical mechanics
- Rigid bodies and their dynamics
- Soft bodies
- Fluids
- Vehicle physics
- Ragdoll physics, Cloth simulation
- Destructible environments
- …

Complex physics models - Reality

- Most of these mechanics are very complex and have little usage in games
- All the games out there with physics use **rigid body dynamics**
- Racing and simulator games also include vehicle physics
- Ragdoll physics is used for death animations of characters
- Soft bodies are used rarely, too complex or not well plugged into a game
  - Added value for the player is usually negligible
- Cloth simulation – mostly for rendering
- Fluid dynamics is usually overkill, animating oceans or water is done using simpler tricks
  - These tricks do not allow the player to interact much with it
  - Portal 2 looks like it's using fluid dynamics, but it's a simple trick as well
  - Terraria, Minecraft use extremely simplified models

Rigid body dynamics

- We will focus only on this part of realistic physics
- Our objects will be **rigid bodies** – non-deformable solid objects
- We want to compute the **dynamics** of rigid bodies
  - The movement and rotation of rigid bodies
  - Based on Newton's laws of motion (a.k.a. dynamics)
- There are also several constraints we can place on objects
  - Static objects – not movable by any force
  - Kinematic objects – moved by the users (such as the player character), but not moved by other objects such as grenades or bullets that hit the character
    - Apply force on other objects, any force applied to them is ignored
  - Joints, connected components (car wheel)…
- We will first go through **linear dynamics**

Moving with constant acceleration

- Movement of the object through the world is a *position function* $X(t)$
- **We do not know the values of $\boldsymbol{X(t)}$ until all player input up until time $\boldsymbol{t}$ is known**
- Constant velocity => $X(t) = X_0 + t * \boldsymbol{v}_0$
- Derivative of the position function is *velocity*
- $\frac{\partial X}{\partial t} X(t) = \boldsymbol{v}(t)$
- 2nd order derivative: *acceleration*
  - When the acceleration is a zero vector, the velocity is a constant vector
  - Otherwise, velocity is a function of time (const. accel.): $\boldsymbol{v}(t) = \boldsymbol{v}_0 + t * \boldsymbol{a}$
  - Or with variable acceleration: $\boldsymbol{v}(t) = \boldsymbol{v}_0 + t * \boldsymbol{a}(t)$

Example

- Parabolic path of a projectile
  - "ballistic shot"
  - Works for all kinds if we ignore air friction
    - Rocks, cannonballs, bullets
- We have an initial velocity $\boldsymbol{v}_0$ and initial position X_0
- Acceleration is equal to gravity, which is a constant
  - $\boldsymbol{a} = (0, -g, 0), g = 9.81ms^{-2}$
- $X(t) = X_0 + t * \boldsymbol{v}_0 + \frac{1}{2}t^2 * \boldsymbol{a}$
- $\boldsymbol{v}(t) = \boldsymbol{v}_0 + t * \boldsymbol{a}$
- $\boldsymbol{a}(t) = \boldsymbol{a}$

Forces

- We want a way to compute the acceleration
- Acceleration is in a relationship with **force**
- $\boldsymbol{f} = m * \boldsymbol{a}$
  - $\boldsymbol{f}$ – force $[N, kg \ * \ m/s^2]$
  - m – mass $[kg]$
  - $\boldsymbol{a}$ – acceleration $[m/s^2]$
- Multiple forces affect a single object, we need to compute the position, velocity and acceleration in the next time step with respect to all those forces
- Gravity – we assume the world is flat
- Air friction
- Contact force

Getting the desired positions

- Forces are 3D vectors (direction + magnitude)
- Adding all forces affecting an object in frame N gives us the combined force that will determine the actual movement of the object
- Computing the result
  1. Apply forces $\boldsymbol{f}(t) = \boldsymbol{f}_0(t) + \boldsymbol{f}_1(t) + \cdots + \boldsymbol{f}_k(t)$
  2. Compute acceleration from forces $\boldsymbol{a}(t) = \frac{\boldsymbol{f}(t)}{m}$
  3. Integrate acceleration to get velocity $\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\, dt$
  4. Integrate velocity to get position $X(t) = \int \boldsymbol{v}(t)\, dt$
  5. Move objects to the desired position $X(t)$

Linear momentum

- We have a relationship between acceleration and velocity:
  - $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$
- The quantity $\boldsymbol{p} = m * \boldsymbol{v}$ is the **linear momentum** and is related to a force $\boldsymbol{f}$
  - $\boldsymbol{f} = m\boldsymbol{a} = m\frac{d\boldsymbol{v}}{dt} = \frac{d\boldsymbol{p}}{dt}$
- *"The tendency of an object to remain in its current linear motion"*
- If the external force of a system is zero, $\boldsymbol{p}$ is constant
  - Very important for handling collisions

Moving with variable acceleration

- Analytic solution for computing the position and velocity is hard in general
  - We work with forces, so we start from acceleration
  - We will not find the exact equation for $\boldsymbol{v}(t)$ and thus not for $X(t)$
- Impluse forces are simple
- Problems occur with variable forces that are applied continuously
  - Objects in contact, friction, springs, joints…
- Numerical integration solves these equations
  - Advanced topic, not covered here
  - See more in book references

Rotational dynamics

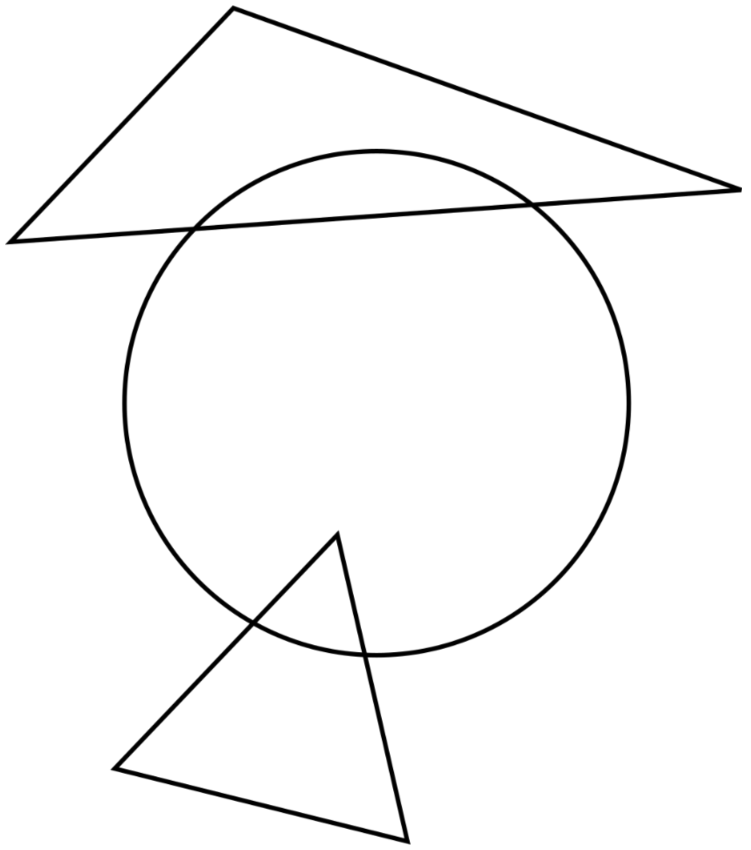
- We have not covered rotation of objects yet!

Position X	$\Rightarrow$	orientation $\boldsymbol{q}$ (a quaternion)
Velocity $\boldsymbol{v}$	$\Rightarrow$	angular velocity ω
Force $\boldsymbol{f}$	$\Rightarrow$	torque τ
Linear momentum $\boldsymbol{p}$	$\Rightarrow$	angular momentum $\boldsymbol{L}$
Mass m	$\Rightarrow$	inertia tensor $\boldsymbol{J}$

- Center of mass important for computing the center of rotation
- The process of computing rotational dynamics is similar to linear dynamics
  - A little more complex, no time to go into detail
- Applying any force to an object is then split into two forces
  - Translational & rotational

Object-object interaction

- So far, we have talked about a single object affected by forces
- What happens when there are more objects?
  - At the moment, objects will pass through each other
- We need to detect when two objects are colliding
- This area is called **collision detection**
- Collision response is the next important part
  - We know two objects are colliding, what now?
  - Objects in contact apply forces to each other



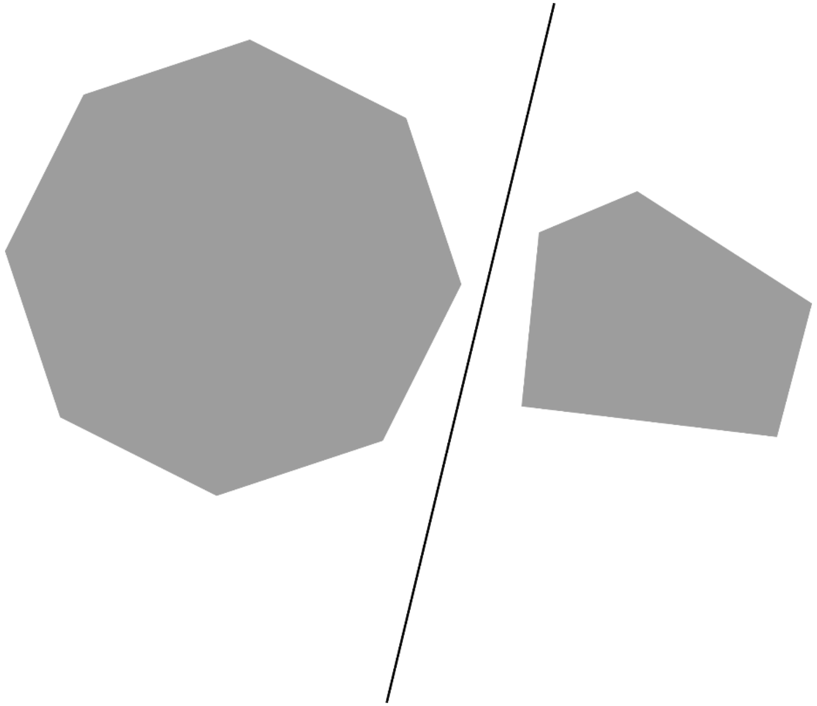
Intersections

- Objects (rigid bodies) are just triangular meshes or simple well-defined shapes (spheres, capsules, boxes…)
- We might use other objects to determine intersections
  - For instance rays when shooting from a gun (or picking an object in 3D)
- Other simple helper objects
  - Planes, Spheres, Capsules, Axis-Aligned Boxes, Oriented Boxes…
- When working with intersections, we might want 2 types of result
  - Simple Boolean telling us if we are intersecting or not (our current focus)
  - The actual intersection (point, triangle, mesh, …)

Determining the intersection

- We have lots of different objects
  - Rays, Triangles, Planes, Spheres, Capsules, Axis-Aligned Boxes, Oriented Boxes…
- To determine intersection between either two, a special algorithm must be used that is designed especially for the two types
- Very easy and fast-to-compute intersection algorithms
  - Ray-Sphere, Ray-Plane, Ray-AAB, Ray-Triangle
  - Sphere-Sphere, Sphere-Plane, Sphere-Triangle
  - AAB-AAB, AAB-Plane, …
  - Fast enough to performs thousands of these per second

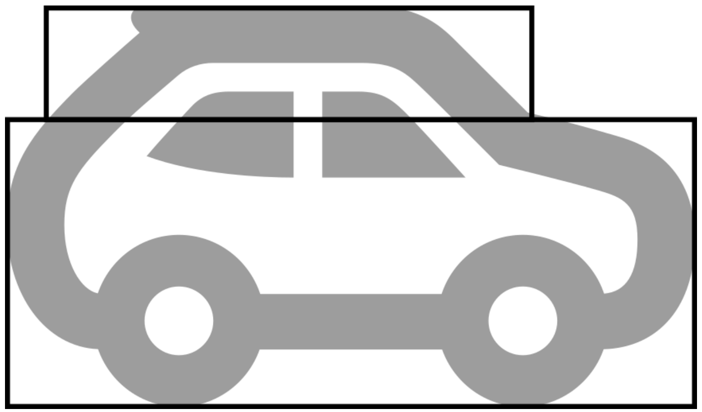
Complex intersections



- Convex meshes
  - Intersections with convex meshes are hard in general
  - Usually, we need to test every single triangle
- Non-convex meshes
  - Even harder than convex meshes
  - Might need to split non-convex meshes into several convex parts for performance reasons
- So slow that only tens to hundreds of tests can be executed per second
  - Heavily depends on mesh complexity
- Mesh-Mesh intersection
  - Might end up testing every triangle-triangle pair $\Rightarrow O(n^2)$
  - Two meshes with 10000+ triangles (not so much for rendering) will take terribly long

Optimizing intersections

- Use simpler shapes
  - Encapsulate complex objects with simpler ones
- No need for exact precision
- Take advantage of hierarchical space-partitioning
- Use bounding volumes

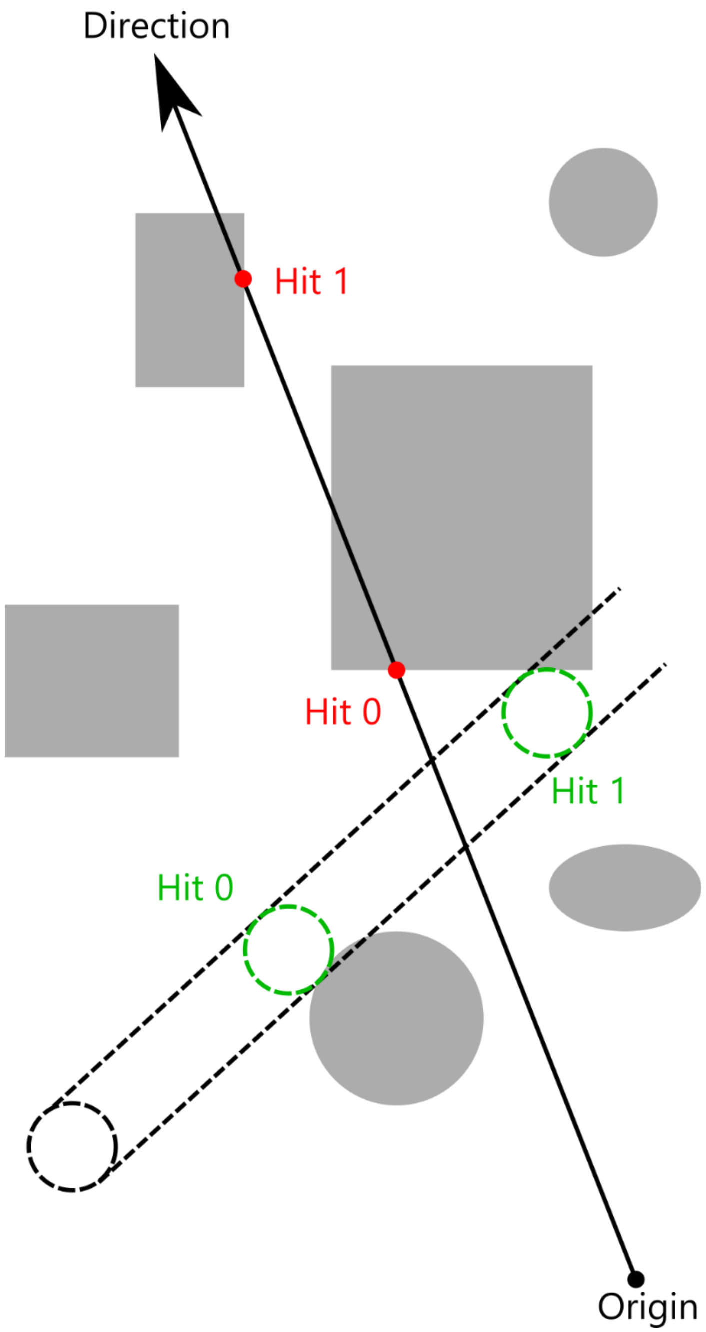


Bounding volumes

- Bound complex mesh geometry with simple objects
- Bounding Spheres
- Axis-Aligned Bounding Boxes (AABB)
- Oriented Bounding Boxes (OBB)
- Use hierarchies of bounding objects for compound objects

(Ray)casting

- Casting an object (ray, sphere…) along a line in a scene to determine which objects are hit and in what order
- Used to determine closest objects along a line
  - Can be used to determine all objects we hit
- We can work with the contact points
- Example: shooting a gun in Counter-Strike
  - Cast a ray from the camera in the viewing direction
  - Determine what we hit first – it's instant (usually called "hit-scan")
  - If it's a character ⇒ deal damage
  - If it's a wall ⇒ create a decal
  - Much more robust than fast moving projectiles



Space partitioning

- If we have thousands of objects in the scene and need to compute object-object intersection for each
- Again, $O(n^2)$ tests
- Create a data structure that allows fast querying of possibly intersecting objects
- Discard distant objects fast
- Possibilities
  - Regular grid – BAD
  - Irregular grid – better
  - Bounding Volume Hierarchies (BVH) – much better
  - BSP or kD-trees - best

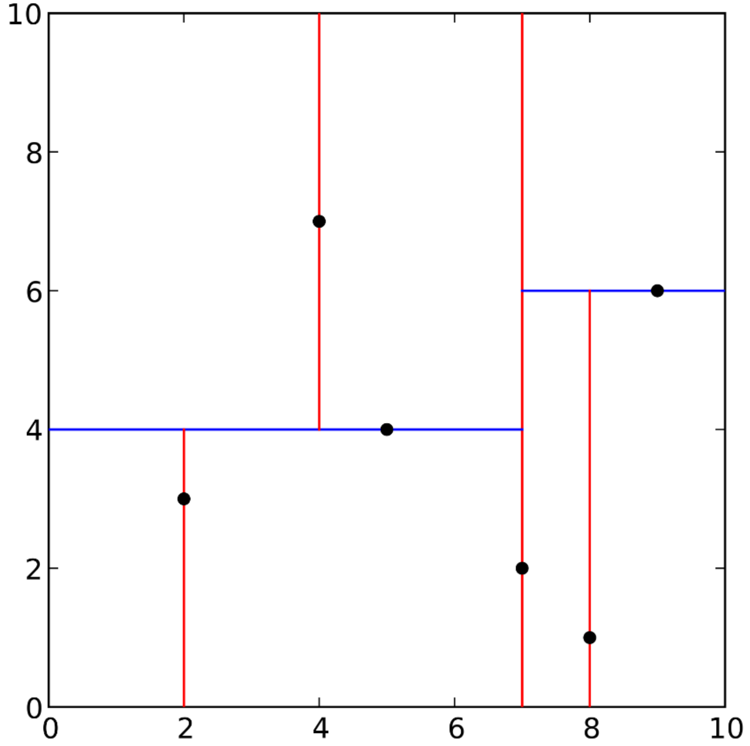
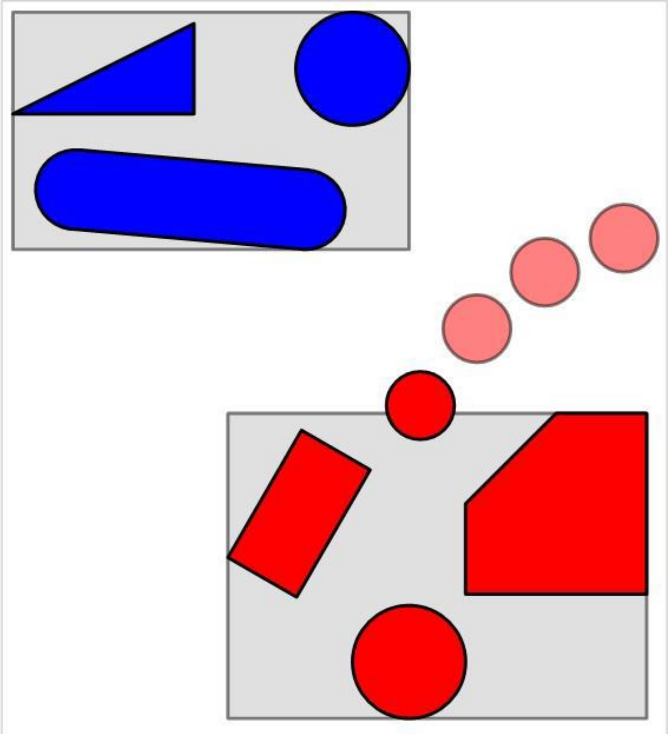


Image from https://en.wikipedia.org/wiki/K-d_tree

Space partitioning (2)

- Need to consider dynamic objects
- Our space partitioning system must allow fast rebuilding of the hierarchy
- Or small adjustments in the tree structure during update
- Worst case – rebuilding the whole tree



Images from http://www.essentialmath.com/GDC2007/1_GDC2007_Eiserloh_Squirrel_PhysicsOverview.ppt

Collision response

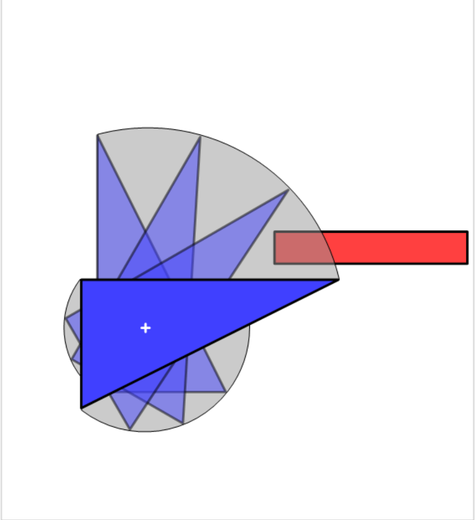
- Once we know two objects are colliding (will be colliding in the next frame), we want to produce a correct response
- Need to consider physical materials
  - Throwing a ball on concrete vs. throwing a ball on soil
  - Friction, bounciness…
- Collision response is produced as additional forces
- A new force is produced for both objects
- For this, we need to determine the actual intersection locations

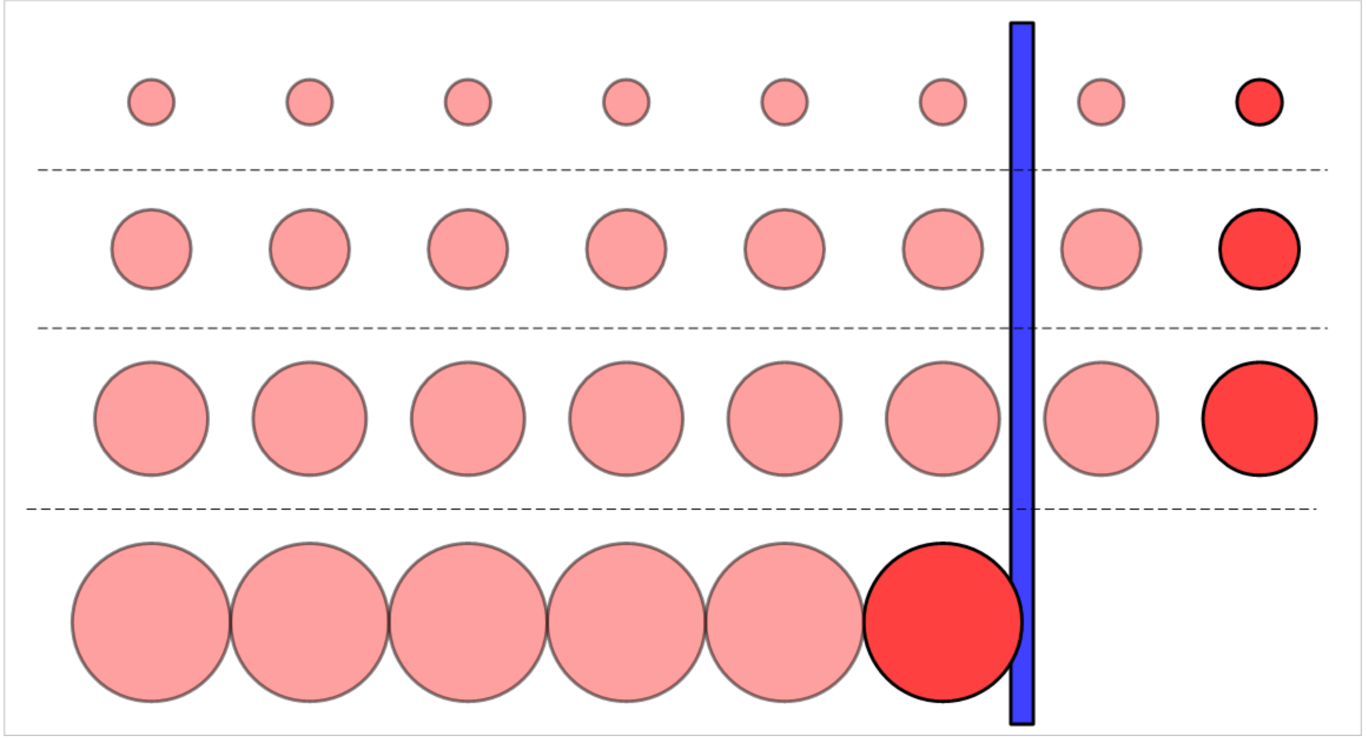
Precision problems

- To be fast enough, we cheat, but create several problems that can occur
- Fixed time updates should occur in infinitely short intervals
  - Otherwise, a fast-moving object might pass through walls, because we didn't catch the moment when it was colliding
- Using static intersections instead of dynamic
  - Using algorithms that consider linear and angular velocity of objects might solve the previous problem
  - However, these algorithms are usually much slower to compute
  - Viable (fast enough) for simple bounding volumes such as spheres
- Solving collision response pair-wise
  - If we solve the collision for A, B and decide to move B so that it collides with C
  - Then, the same happens for B, C => A; C, A => B…

Tunneling

- Small & fast objects go through other objects
- Solutions:
  - Minimum object size
  - Maximum speed
  - Simulate more often
  - Ideally combine all 3





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Solutions

- Ostrich approach
  - Ignore the problems and let’s hope a bug will never occur
- Selective approach
  - Select dynamic collision algorithms for more important and fast-moving objects
  - Grenades, bullets, …
  - Any object it will collide with is going to have to use the dynamic intersection version
  - Or at least the “half-dynamic” version (one object dynamic, the other static)

- Know physics limitations
  - Thickness of colliders & maximum object speed
  - Amount of objects
  - Complexity of objects – colliders, rigidbodies, joints…

Unity's physics engine

- The Transform component of a game objects contains the position X and orientation q
- We add physics through components
- **Rigidbody** - Provides *mass* for our object and allows forces to affect it
  - Other settings such as *drag, collision detection settings, kinematic settings…*
- **Colliders** - Give our objects *shape*
  - Allow us to detect *collision events*
  - Can be triggers that do not actually collide but provide *trigger events*
    - "regions" we can activate
  - Are used to calculate the inertia tensor automatically
- **Joints** and **cloth** - Constraints and non-rigid simulation
- **Character controller** – unaffected by forces, responds to collisions
  - Special component for controllable characters – physically unrealistic
- NVIDIA PhysX (3D), Box2D (2D), Havok (experimental 3D replacement)

Unity's physics engine scripting

- Physics computation happens after `FixedUpdate`
- The Rigidbody component allows for manipulation of objects with forces
  - Hitting objects with colliders produces contact force
  - Computes center of mass automatically
  - `AddForce`, `AddTorque`, `AddForceAtPosition`
  - Manually setting velocity – possible but not recommended
  - Can have physical material (friction and bounciness)
- Collider components
  - Can have density (for Rigidbody auto-mass computation)
  - Can be triggers
  - Provide collision callbacks
    - `OnCollisionEnter`, `OnCollisionStay`, `OnCollisionExit`
    - `OnTriggerEnter`, `OnTriggerStay`, `OnTriggerExit`

Execution Order of Event functions

- What Unity events get called in what order
- Most important image for every Unity programmer!!!

https://docs.unity3d.com/Manual/ExecutionOrder.html

Collisions messages called only in some cases

https://docs.unity3d.com/Manual/CollidersOverview.html

Collision detection occurs and messages are sent upon collision						
	Static Collider	Rigidbody Collider	Kinematic Rigidbody Collider	Static Trigger Collider	Rigidbody Trigger Collider	Kinematic Rigidbody Trigger Collider
Static Collider		Y				
Rigidbody Collider	Y	Y	Y			
Kinematic Rigidbody Collider		Y				
Static Trigger Collider						
Rigidbody Trigger Collider						
Kinematic Rigidbody Trigger Collider						

Trigger messages are sent upon collision						
	Static Collider	Rigidbody Collider	Kinematic Rigidbody Collider	Static Trigger Collider	Rigidbody Trigger Collider	Kinematic Rigidbody Trigger Collider
Static Collider					Y	Y
Rigidbody Collider				Y	Y	Y
Kinematic Rigidbody Collider				Y	Y	Y
Static Trigger Collider		Y	Y		Y	Y
Rigidbody Trigger Collider	Y	Y	Y	Y	Y	Y
Kinematic Rigidbody Trigger Collider	Y	Y	Y	Y	Y	Y

Unity’s physics engine scripting (2)

- Physics class
  - Global physics settings (As well as Project Settings => Physics)
  - Overlap tests
  - Cast tests
  - Closest point computation

References

- http://www.essentialmath.com/tutorial.htm
- Mathematics for 3D Game Programming and Computer Graphics
- Essential Mathematics for Games and Interactive Applications: A Programmer's Guide